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New Ways to Handle Spatial Relations through Angle plus MBR Theory on Raster Documents

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Abstract

The paper presents novel ideas on spatial reasoning by considering shape and size of spatial entities and focuses on extension of conventional models: Cone-Shaped and Minimum Boundary Rectangle (MBR). Firstly, it goes on the use of how well MBR features fixed to cone-shaped model considering the relative extension of the spatial entities. This suggests the further possibility and the quality of combinatorial study of the models in spatial reasoning. Secondly, another model is proposed by referencing a unique common point set (of two entities) instead of using one out of two entities (referencing in a conventional way). This makes model robust and more symmetric as it reserves two way relationships from the unique reference point set. Further, it gives flexibility to utilize conventional models. Finally, it gives a comparative idea of proposed models among existing ones.

Keywords: Angle based Theory, MBR, Hybrid Model, Unique Reference Point Set

1 Introduction

Spatial information between parts of images has been an important element for graphical symbol recognition methods. It has been an important clue towards learning, identifying and interpreting the contents. Therefore, decomposition of a complete shape into a number of possible components has been one of the approaches to describe symbols and other visual patterns in many recognition tasks (Fig. 1 is one of the examples [1], [2]). However, a variety of potential errors arising from the randomness, incompleteness and vagueness of the spatial data, limits the complete usefulness of the spatial reasoning, since the result of automatic methods extracting spatial relations tend to be very sensitive to the shape and the position of the components, and become easily affected by these potential errors. This eventually affects the global confidence in recognition.

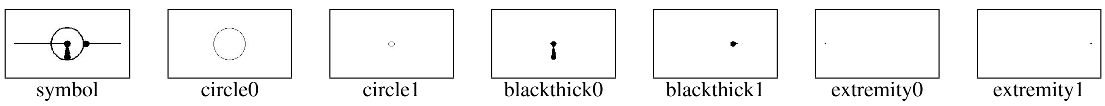


Figure 1: Possible set of components generated from a symbol

Spatial reasoning comes in two main classes: qualitative and quantitative. Qualitative representation is based on comparative knowledge rather than on metric information [3]. Basically, interpretation of qualitative spatial relations result in natural language-like descriptions where precision is not always necessary. Furthermore, qualitative knowledge is usually computationally cheap to obtain. On the other hand, quantitative representations are often based on fuzzy set theory [4] and tend to allow for a better management of the ambiguities that arise on border conditions.

Fundamental work by Freeman *et al.* [5, 6]) divides spatial relations into three families: topological (that describe neighborhood and incidence e.g. *Dis-Connected, Externally Connected...*), metric (e.g. *Near, Far...*) and directional (that describe order in space e.g. *Left, Right...*). This paper mainly focuses on topological and directional relations. However, the proposed approaches may certainly affect the way to determine the metric relations as well. This aspect is currently beyond the scope of our work.

The goal of this paper is to give a comprehensive overview of different approaches to computing spatial relations between objects in an image and to compare them with respect to intuitive human perception.

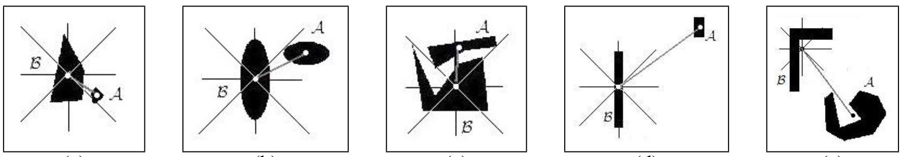
The paper is organized as follows. In section 2, we describe existing approaches to computing spatial relations in images and analyze their mutual advantages and drawbacks. Next, in section 3 we develop a new method, combining elements from the previously studied work, in order to enhance or correct them. Sections 4 and 5 describe in detail our experimental setup and assess the improvements of our approach. Section 6 concludes the paper along with few future directions to move.

2 State-of-the-Art

This section starts with few outlines and straightforward drawbacks of existing approaches. Every model is found to be problem dependent and varied according to the following parameters: spatial relationships, environment and the behavior of the objects to be studied. Therefore, one has to extend the model according to the problem.

2.1 Cone-Shaped Model

The cone-shaped or centroid angle theory is the first to be considered. It is based on the angle made by a centroid of primary object with respect to reference object ($\angle(C_P, C_R)$)(Fig.2). The angle value is then discretised. The preciseness of cardinal relationship is determined by star calculus [7, 8]. This method is robust to small variations of shapes and sizes but, it does not cover information about objects' shapes and their separation. This is particularly noticeable with concave objects. In such a concavity case, the centroids may not fall within the objects and the model may lead to the detection of a wrong direction. But, on the other hand, if objects are very far from each other, the cone-shaped model works well because objects can be assumed to be points. Furthermore, it is limited to where two centroid are confounded since no angle value can be computed. This is a very common situation.



(a) (b) (c) (d) (e)

Figure 2: Simple concept on cone-shaped model based by using four quadrants

A very similar work with the basic theory is presented by Koji *et* al, 1994 [9]. However, an ambiguity between pair of objects persists as shown in Fig. 3 (a). In addition, the use of centroid angle is not a perfect solution for the examples shown in Fig.3 (b). In case of the first pair of objects in Fig. 3 (b), the cone-shaped model locates $\mathbb{A}$ on top of (or above) $\mathbb{B}$)', which is not quite relevant: it seems to be to the *Left*. Moreover, if objects are connected and overlapping, the cone-shaped model is less suited and the MBR yields better results. The MBR is described in the following section.

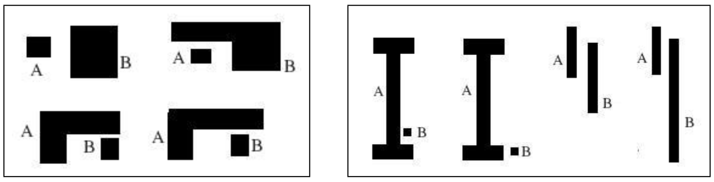


Figure 3: (a) Ambiguity on $\mathbb{A}$ is left of $\mathbb{B}$ (b) Cone-shaped model does not yield relevant result

2.2 MBR Model

MBR [10, 11, 12, 13] gives more interesting results compared to the cone-shaped model. Externally aligned orientations through MBR are *Left*, *Right*, *Top* and *Bottom* etcetera regardless the shape of the objects. Those relationships are straightforwardly derived from a partitioning of the surrounding space based on the lines supporting the sides of the rectangle, as shown in Fig. 4. For more clear understanding, four boundary lines sketch the plane for each region: *Left*, *Right*, *Top*, *Bottom* and so on.

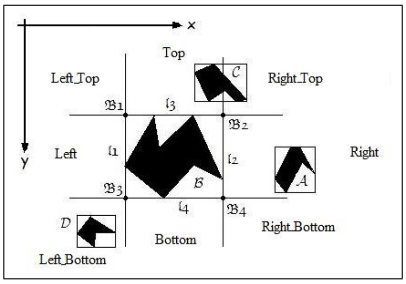


Figure 4: Classical MBR model

Compactness of the MBR tile is an important parameter to be considered[1]. The quality encasing rectangle depends directly on the shape of the object and eventually related to compactness of the tile. Possible configurations of relative positioning with MBR are demonstrated in Fig. 5. There are 218 possible relations between non-empty and connected regions (Sun *et* al., 2007 [13]). The sole information used in the MBR approaches is derived from the geometry of the bounding rectangle. The definition of directional spatial relationships between rectangles is due to Peuquet *et* al., 1987 [14], and was widely adopted since [15, 16, 17]. A bi-dimensional object is represented as a set of couple of perpendicular segments and ultimately, spatial relations are deduced from relations between segments [18, 19]. Improvements have been proposed to affine spatial representations. For instance, Petry *et* al., 2002 [20] proposed a partitioning into a set of rectangles to embed the shape but the approach relies on scale thresholds and continuous assessment is not warranted. In addition, MBR approximates topological relations in spatially extended objects, which in turn may express false connection/overlapping.

2.2.1 Topology

Topological relations through MBR [11] are demonstrated in Fig. 6. Based on this, a 9-intersection model [21] is used for binary topological relations,

$$Topology(\mathbb{A}, \mathbb{B}) = \begin{bmatrix} \mathbb{A}^o \cap \mathbb{B}^o & \mathbb{A}^o \cap \partial\mathbb{B} & \mathbb{A}^o \cap \mathbb{B}^- \\ \partial\mathbb{A} \cap \mathbb{B}^o & \partial\mathbb{A} \cap \partial\mathbb{B} & \partial\mathbb{A} \cap \mathbb{B}^- \\ \mathbb{A}^- \cap \mathbb{B}^o & \mathbb{A}^- \cap \partial\mathbb{B} & \mathbb{A}^- \cap \mathbb{B}^- \end{bmatrix}$$

[1] Simply, compactness is defined as the ratio of area of an object to the area of the encasing rectangle. It lies within the range: $0 \leq Compactness \leq 1$.

where $\mathbb{A}^o$, $\partial\mathbb{A}$ and $\mathbb{A}^-$ are the interior, the boundary and the exterior for a given point set (object) $\mathbb{A}$. ***Pixel Wise Matching*** *(PWM)* is introduced for the refinement of false overlapping/connection that may occur by considering only the MBR, which is sometimes known by *intersection* of objects: $\mathbb{A} \cap \mathbb{B}$. This is quite similar to RCC-8 [12]. Load (time complexity) linearly follows the size of the studied objects.

Including the effect on directional and topological relations, MBR has found to be better because of its possibility in accounting additional directional metric information [22].

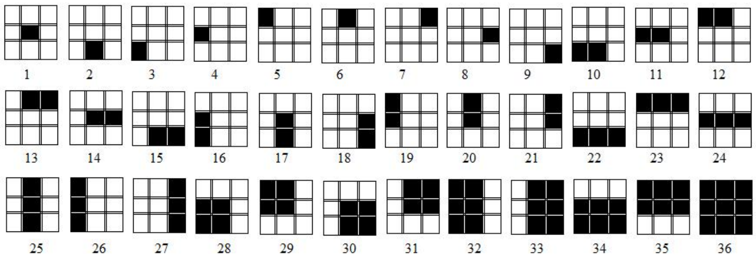


Figure 5: Possible configurations (36) through MBRs (use of MBR in both objects)

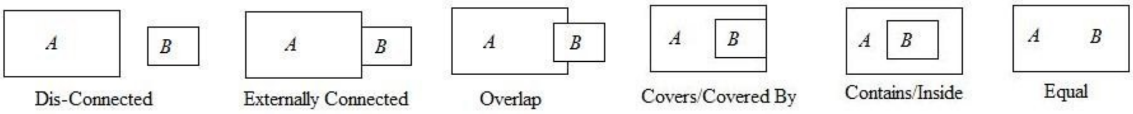


Figure 6: Illustrations of topological relations through MBR

2.3 Angle Histogram

Overlapping is a complex problem and approaches based on angle histogram seems to be more efficient than MBR in this configuration. For objects $\mathbb{A}$ and $\mathbb{B}$, sets of coordinates are: $\mathbb{A} = \sum_{i=1}^{m} a_i$ and $\mathbb{B} = \sum_{j=1}^{n} b_j$. $a_i = [x_i, y_i]$ and $b_j = [x_j, y_j]$. We have $m \times n$ pairs of points and collection of angles $\theta_{i,j} = \angle\angle(a_i, b_j)$. The histogram of angle can be considered as $H_\theta(\mathbb{A}, \mathbb{B}) = [\theta, f_\theta]$. Both the centroid and the aggregation angle methods are mainly based on a single value. The major difference is that the averaging is made on the object's points for the centroid method while it is applied after angle computation in the aggregation method. Briefly, there is no difference between the cone-shaped and the angle histogram model when two objects are relatively at large distance since objects can be assumed to be a point. In such a case, time complexity is only the issue. On the other hand, it is difficult to generalize 3D as it needs two angles to define a position of a line with respect to coordinates axis and thereby it increases the load.

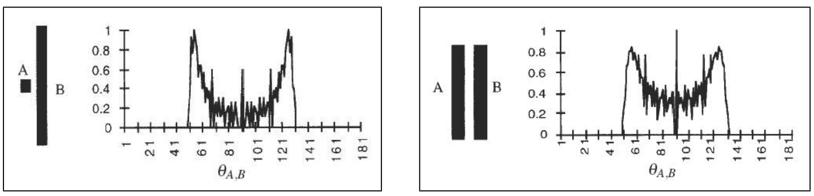


Figure 7: Image and angle histogram (Wang *et* al., 1999)

The major inaccuracy of the model is examined by Wang *et* al., 1999 [23]. Inaccuracy here refers to similarity of histogram of angles as shown in Fig. 7. Due to this, more features have to be added in order to separate those pairs of images. In order to decrease the load of the histogram of angles, boundary of the regions have been taken instead of all the pixels within it. But, it does not give an accurate angular direction (because of non-uniform density of pixels over the regions). On the way, Wang it et al., 2003 proposed metric information [24] in addition to histogram of angles, i.e., R-histogram. But, complex objects with holes cannot not be treated suitably. Therefore, extra works have to be done. For a quick instance, the distance from the primary object and a secondary object resides within the hole of the primary object is found to be complicated.

2.4 Force Histogram (F-Histogram) and F-Templates

The most robust is method based on force histograms and gives coherent results in any configuration (Matsakis *et* al., 1999 [25]). This method is generic and depends on a robust mathematical framework. It considers pairs of longitudinal sections instead of pairs of points. Nonetheless, processing time is high because of the number of processed directions while considering particular field of raster documents where shape are rather regular. Moreover, for topological relations such as, *Inside* and *Overlap*, and for metric information, it requires extra interpretations. Furthermore, Matsakis *et* al., 2006 [26] and Wang *et* al., 2006 [27] introduced new concept of F-Templates in order to receive algorithm faster with better results. Besides, it is more flexible as it approximates new algorithm into F-histogram. But, F-templates theory tends to give identical results among the pairs of objects though reference object has been changing its shape from one pair to another, which is shown in Fig. 8. Besides, a single point places an effect on overall results.

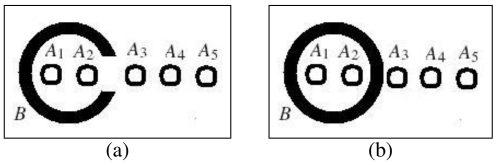


Figure 8: F-template yields identical directional relationship for all pairs of objects ($\mathbb{B}$ - Reference)

3 Proposed Approach

Considering those drawback issues, we propose new ways to handle spatial relations that certainly outperform conventional models. The paper presents two steps of advancement. Firstly, Extended MBR (X-MBR) model is proposed which addresses preciseness of spatial orientations compared to classical MBR (section 3.1). Secondly, hybrid models are discovered. In the first stage, the cone-shaped has been advancing according to the X-MBR structure (see section 3.2.1). This allows for the size of the object to be taken into account. While in the second stage, we introduce a unique reference point set (common part of two entities – see section 3.2.2). This introduces symmetric features in generating spatial relationships. This unique reference point set can be a region, a line or simply a point. This is entirely based on organization of topological relations. In such a unique reference point set, two extra extended approaches: extended angle based and MBR theories are applied. Topological relations of MBR are fixed with the help of intersection model [21].

3.1 Step 1: Extended MBR (X-MBR) Model

Within the MBR, the centroid is an important clue for generating precise spatial relationships. It is demonstrated in Fig. 9, where the centroid is implemented for splitting a particular spatial plane into two. For instance, *Right*

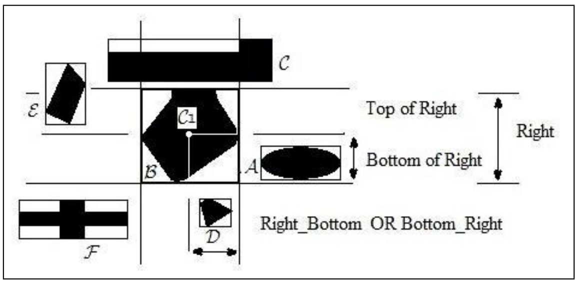


Figure 9: An Extended MBR (X-MBR) model

can be sub-divided into *Top of Right* and *Bottom of Right* (see Fig 9). This can be possible for other three regions: *Top*, *Bottom* and *Left*. But, *Right_Top*, *Right_Bottom*, *Left_Top* and *Left_Bottom* are used as usual. This is referred to as extended MBR (X-MBR) model. Besides binary relationships, extension of any target objects can be precisely achieved with the help of following ideas [22]. This registers how much of the target object falls into in each directional space.

$$Dir(\mathbb{A},\mathbb{B}) = \begin{bmatrix} \frac{area(Left\_Top_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(Top_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(Right\_Top_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} \\ \frac{area(Left_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(M_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(Right_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} \\ \frac{area(Left\_Bottom_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(Bottom_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} & \frac{area(Right\_Bottom_{\mathbb{B}} \cap \mathbb{A})}{area(\mathbb{A})} \end{bmatrix}$$

3.2 Step 2: Towards Hybrid Models

Hybrid model, here refers to the combinatorial study of conventional models - Cone shaped and MBR in order to enhance the quality of spatial relationships under the same scope. This gives the realistic concept of how we can improve the existing models such that the extended models are able to recover those common problems (section 2).

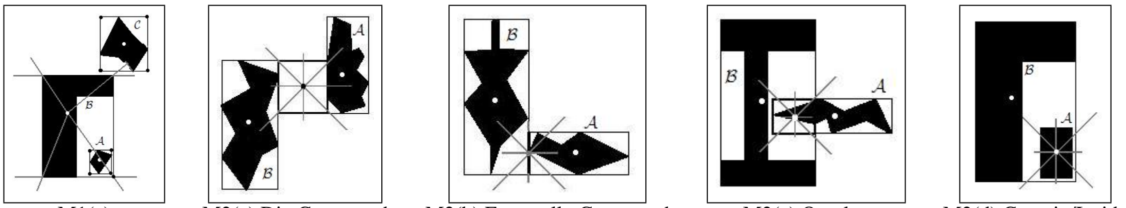
3.2.1 M1: Dynamic Angle Based Theory by Using Line via Every Corner of Rectangle from center

An advancement of cone-shaped model with the inclusion of MBR structure is discovered for generating coverage plane according to the object's size. This theory suggests that the range of angle made by lines drawn from centroid to every corner of MBR directly depends on the shape and size of rectangle. Therefore, it is referred to as dynamic angle based theory. In Fig. 10, M1(a) illustrates the dynamic coverage plane in all directions which depends on the shape of the MBR. It illustrates the way of how we are able to recover the confusions reported in Fig. 3 (b) and Fig. 7. With this idea, eight directional relations is taken by using two narrowed sides of the MBR in order to cut off the region for additional four directional planes (corners' sides). Further, the model takes level of centroid that aids preciseness as discussed in the X-MBR model (section 3.1).

3.2.2 M2: Extended Model by Using a Unique Reference Point set

Reference is always an important factor in spatial reasoning. Which object would be the best choice for referencing is a common problem. This is what the model attempts to explore. A unique reference point set from a common portion of two entities is taken for two way relationships rather than referencing one out of the two.

In Fig. 10, M2(a-d) gives a simple demonstration of *Dis-Connected*, *Externally Connected*, *Overlapped* and *Contain/Inside*. The basic idea of the model is easily explained from the topological formation. In case of *Dis-Connected* objects, the reference region is determined from a common region, through orthogonal projection of the two MBRs onto the image axes. In case of *Externally Connected* components, the common portion



M1(a) M2(a) Dis-Connected M2(b) Externally Connected M2(c) Overlap M2(d) Contain/Inside

Figure 10: M1 (a): Dynamic angle based theory by using lines passing through every corner of the MBR from center and M2 (a-d): Extended model by using a unique reference point set

is just a line and sometimes a point. In a similar way, the portion of overlapping is a reference region for *Overlap*ping case, while *Cover*ed/*Contain*ed component is becoming a reference region for both *Cover/Covered By* and *Contain/Inside* cases. The common reference region gives the means of fixing the distance computation between objects in case of disjoint regions. It does not matter if the common region is only a unique point or a line, since centroid is is used. Therefore, the model is not limited to the use of X-MBR model but, it can be extended to the use of point to region angle based theory.

4 Experimental Setup

The basic procedure includes, noise removal, segmentation, labeling, geometrical interpretation extraction and automatic semantic spatial relationship generation. All experiments were done by using MATLAB 7.0.1 on a 2.40GHz, 3.00GB RAM and 32 bit operating system, running Windows Vista. In order to prove the proposed idea, two types tests were done separately for global spatial objects of different shapes and sizes and components generated from electrical graphical symbols.

Spatial Prepositions Use: Under the scope of the spatial reasoning world, very common and natural way to share spatial relations is through the use of spatial prepositions [28] (Schmidt, 1988) in order to describe the relationships between the spatial entities. Human interpretation [23, 29] is important and similarly spatial relations are also coming from linguistic and psychology fields [6, 28]. In this paper, the following index gives overall use of spatial predicates in order to grasp consistent understanding.
Topological Relationship: DC = *Dis-Connected*, EC = *E*xternally Connected, O = *Overlapped*, Cr = *Cover*, CB = *Covered By*, Ct = *Contain*, I = *Inside* and EQ = *Equal*
Directional Relationship: R = *Right*, L = *Left*, T = *Top*, B = *Bottom*, R_T = *Right_Top*, R_B = *Right_Bottom*, L_T = *Left_Top*, L_B = *Left_Bottom*, BoR = *Bottom of Right*, ToR = *Top of Right*, BoL = *Bottom of Left*, ToL = *Top of Left*, RoT = *Right of Top*, LoT = *Left of Top*, RoB = *Right of Bottom*, LoB = *Left of Bottom*
Others: $\mathbb{X}$-Ref = $\mathbb{X}$ is the reference $\mathbb{R}p$ = Reference point, c_x, c_y = centroid of $\mathbb{X}$ and $\mathbb{Y}$ entities
Example: Relationship($\mathbb{X}$,$\mathbb{Y}$) = ToR:R_T($\mathbb{X}$,$\mathbb{Y}$) means $\mathbb{X}$ is extended from *ToR* to *R_T* with respect to $\mathbb{Y}$

5 Assessment and Discussion

Assessments start with cone-shaped, MBR and follow X-MBR model for precise pairwise relationship. In the second phase, hybrid models are studied including simple point to point and point to region plus advancement of cone-shaped and MBR approaches. Point to line is not included here as it comes in between. Region resembles MBR throughout the experiment. In case of M1, topological information have been carried out together with the directional relations. While in M2, topology has been checked in the first step and then heading to generate directional relations. Therefore, relationships are entirely based on topology. Thus, topological relations are

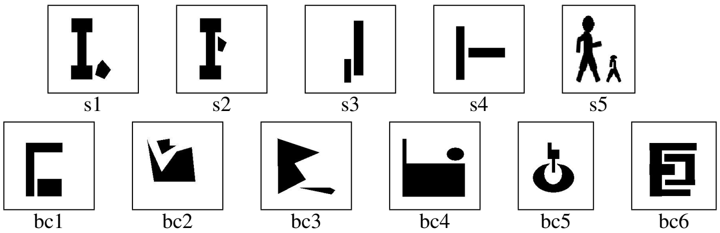


Figure 11: s1:s5 and bc1:bc6 - simple, and broad and complex configurations

not counted in the experimental table only in case of M2. Remember, X-MBR has identical performance with classical one if one has to take topological relations.

Table 1: Spatial Relations through conventional approaches including X-MBR

Image	Cone-shaped	MBB	X-MBB plus PWM
s1	R_B($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	BoR($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
s2	R($\mathbb{A}, \mathbb{B}$)	O($\mathbb{A}, \mathbb{B}$) (Open-R)	O($\mathbb{A}, \mathbb{B}$) (Open-R), DC($\mathbb{A}, \mathbb{B}$)
s3	L_B($\mathbb{A}, \mathbb{B}$)	L:B_L($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	BoL:B_L($\mathbb{A}, \mathbb{B}$),DC($\mathbb{A}, \mathbb{B}$)
s4	R($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
s5	R_B($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	BoR($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
bc1	R_B($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
bc2	L_T($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$) on T, DC($\mathbb{A}, \mathbb{B}$)
bc3	R_B($\mathbb{A}, \mathbb{B}$)	O($\mathbb{A}, \mathbb{B}$) (Open-R)	O($\mathbb{A}, \mathbb{B}$) from BoR, DC($\mathbb{A}, \mathbb{B}$)
bc4	R_T($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$)	CB(x,y) on ToR and RoT, DC($\mathbb{A}, \mathbb{B}$)
bc5	T($\mathbb{A}, \mathbb{B}$)	O($\mathbb{A}, \mathbb{B}$) (Open-T)	O($\mathbb{A}, \mathbb{B}$) from T, DC($\mathbb{A}, \mathbb{B}$)
bc6	$c\_\mathbb{A} = c\_\mathbb{B}$	I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$) in $c\_\mathbb{B}$, DC($\mathbb{A}, \mathbb{B}$)

5.1 Global Objects

Binary pairwise relationship using the cone-shaped model is introduced in Table 1, in which classical MBR and X-MBR are also included. It is straightforward to notice that the MBR yields "better" results (*i.e.* more intuitively acceptable) over the cone-shaped model in every configuration in Fig. 11. A distinguishable feature of X-MBR over classical MBR can be seen with the help of relationships coming from images: s1, s2, s3, s5, bc4 and bc6 in Table 1. For a quick understanding, X-MBR conveys BoR($\mathbb{A}, \mathbb{B}$) from image s1 instead of only R($\mathbb{A}, \mathbb{B}$) from MBR. Similarly, PWM gives correct information about the overlapping of two entities while MBR itself produces false overlapping. In case of configuration where $c\_\mathbb{A} = c\_\mathbb{B}$ (image bc6), X-MBR yields relationships: I($\mathbb{A}, \mathbb{B}$) in $c\_\mathbb{B}$. Table 2 demonstrates the use of M1 model. Within point to point, two different relationships have been discovered by using two types of centroids: the object's centroid and the MBR centroid. It shows how well the object's centroid fits to the model. Point to region gives more information than point to point (for example, results from images s2, bc1, bc2, bc3, bc4 and bc6 would be better examples to compare). In Table 3, M2 shows the use of extended concept of cone-shaped and MBR model via an application of a unique reference region/line/point separately. For this model, few important parameters are needed to sort out before analyzing the relationships. Firstly, in case of angle theory approach, $\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$ means $c_{\mathbb{A}}$ is taken as a unique point from an object $\mathbb{A}$. Similarly, $\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$ indicates a reference region equal to the MBR of $\mathbb{A}$, such that $EQ(\mathbb{A}, \mathbb{R}g)$. According to the recently mentioned comparison between extended angle theory and X-MBR, M2 shows its greater efficiency. No matter which approach has been used, the model shows the usefulness of common regions for distance determination if and only if two spatial entities

Table 2: Spatial Relations through M1 (Relationship(x,y) with MBR centroid/Relationship(x,y) with Object centroid)

Image	Point to Point	Point to Region
s1	BoR($\mathbb{A}, \mathbb{B}$)	BoR($\mathbb{A}, \mathbb{B}$)
s2	R($\mathbb{A}, \mathbb{B}$)	O($\mathbb{A}, \mathbb{B}$), R($\mathbb{A}, \mathbb{B}$)
s3	BoL($\mathbb{A}, \mathbb{B}$)	BoL:L_B($\mathbb{A}, \mathbb{B}$)
s4	R($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$)
s5	BoR($\mathbb{A}, \mathbb{B}$)	BoR($\mathbb{A}, \mathbb{B}$)
bc1	B($\mathbb{A}, \mathbb{B}$)/BoR($\mathbb{A}, \mathbb{B}$)	R_B($\mathbb{A}, \mathbb{B}$) (Large part in B)/BoR($\mathbb{A}, \mathbb{B}$)
bc2	T($\mathbb{A}, \mathbb{B}$)/ToL($\mathbb{A}, \mathbb{B}$)	ToL:T($\mathbb{A}, \mathbb{B}$), CB($\mathbb{A}, \mathbb{B}$)
bc3	BoR($\mathbb{A}, \mathbb{B}$)	B:BoR($\mathbb{A}, \mathbb{B}$) (Large part in B), O($\mathbb{A}, \mathbb{B}$)
bc4	ToR($\mathbb{A}, \mathbb{B}$)	ToR:R_T($\mathbb{A}, \mathbb{B}$), CB($\mathbb{A}, \mathbb{B}$)
bc5	T($\mathbb{A}, \mathbb{B}$)	T($\mathbb{A}, \mathbb{B}$), O($\mathbb{A}, \mathbb{B}$)
bc6	R($\mathbb{A}, \mathbb{B}$)/ I($\mathbb{A}, \mathbb{B}$), $c\_\mathbb{A} = c\_\mathbb{B}$	R($\mathbb{A}, \mathbb{B}$), T:B($\mathbb{A}, \mathbb{B}$)/T:B($\mathbb{A}, \mathbb{B}$), I($\mathbb{A}, \mathbb{B}$)

Table 3: Spatial Relations thorough M2

Image	Use of Ref. Point (Cone-shaped)	Use of Ref. Region (MBR)
s1	R($\mathbb{A}, \mathbb{R}p$), L_T($\mathbb{B}, \mathbb{R}p$)	R($\mathbb{A}, \mathbb{R}g$), L:L_T($\mathbb{B}, \mathbb{R}g$)
s2	R($\mathbb{A}, \mathbb{R}p$), L($\mathbb{B}, \mathbb{R}p$)	R($\mathbb{A}, \mathbb{R}g$), B:L_B:L_T:T($\mathbb{B}, \mathbb{R}g$)
s3	L_B($\mathbb{A}, \mathbb{R}p$), T($\mathbb{B}, \mathbb{R}p$)	L:L_B($\mathbb{A}, \mathbb{R}g$), R:R_T($\mathbb{B}, \mathbb{R}g$)
s4	R($\mathbb{A}, \mathbb{R}p$), L($\mathbb{B}, \mathbb{R}p$)	R($\mathbb{A}, \mathbb{R}g$), L_B:L_T($\mathbb{B}, \mathbb{R}g$)
s5	R($\mathbb{A}, \mathbb{R}p$), L_T($\mathbb{B}, \mathbb{R}p$)	R($\mathbb{A}, \mathbb{R}g$), L:L_T($\mathbb{B}, \mathbb{R}g$)
bc1	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, T($\mathbb{B}, \mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, L:L_T:T($\mathbb{B}, \mathbb{R}g$)
bc2	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, B($\mathbb{B}, \mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, L:L_B:R_B:R($\mathbb{B}, \mathbb{R}g$)
bc3	R($\mathbb{A}, \mathbb{R}p$), L_T($\mathbb{B}, \mathbb{R}p$)	O($\mathbb{A}, \mathbb{R}g$) (Open-R), L:L_T:T($\mathbb{B}, \mathbb{R}g$)
bc4	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, L_B($\mathbb{B}, \mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, B:L_B:L_T:T($\mathbb{B}, \mathbb{R}g$)
bc5	T($\mathbb{A}, \mathbb{R}p$), B($\mathbb{B}, \mathbb{R}p$)	O($\mathbb{A}, \mathbb{R}g$) (Open-T), L:L_B:R_B:R($\mathbb{B}, \mathbb{R}g$)
bc6	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, Ct($\mathbb{B}, \mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, B:L_B:L_T:T($\mathbb{B}, \mathbb{R}g$)

are separated.

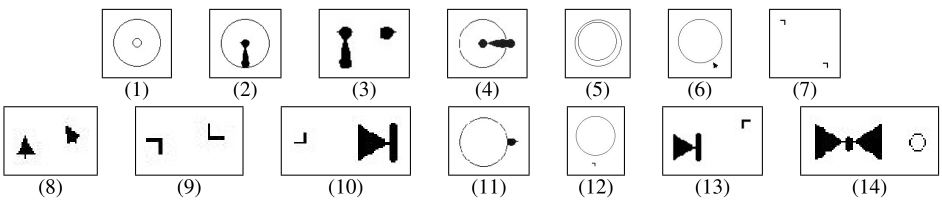


Figure 12: Randomly selected components generated from electrical graphical symbols (Refer to Fig. 1)

5.2 Components of Electrical Graphical Symbols

As in the previous experiment, cone-shaped, classical MBR and X-MBR model are demonstrated in Table 4. Through cone-shaped, the relationship between two hollow circles having same centroid results to nothing (image 1) as the angle cannot be computed. Further, it conveys false directional relationship for a component inside a hollow circle without giving any ideas of topological relations (image 2 and 5). In between two corners (close to same size), it provides interesting relationships. Here, compactness comes to be the worst case, however, MBR works well because of its regularity with respect to size and separation. As mentioned earlier, the major difference between MBR and X-MBR is lying on the precision of spatial relationships (image 2, 3, 8 and 10). Similar problems about the actual connectivity/overlapping, are fixed with the help of PWM at the risk

Table 4: Spatial Relations through conventional approaches including X-MBR

Image	Cone-shaped	MBB	X-MBB plus PWM
1	$c_{\mathbb{A}} = c_{\mathbb{B}}$	I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$), $c_{\mathbb{A}} = c_{\mathbb{B}}$, DC($\mathbb{A}, \mathbb{B}$)
2	B($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$)	CB($\mathbb{A}, \mathbb{B}$) below $c_{\mathbb{B}}$
3	R($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$) DC($\mathbb{A}, \mathbb{B}$)	ToR($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
4	R($\mathbb{A}, \mathbb{B}$)	O($\mathbb{A}, \mathbb{B}$) (Open-R)	O($\mathbb{A}, \mathbb{B}$) (Open-R)
5	T($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$) Above($c_{\mathbb{A}}, c_{\mathbb{B}}$), DC($\mathbb{A}, \mathbb{B}$)
6	R_B($\mathbb{A}, \mathbb{B}$)	B($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)	RoB($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
7	L_T($\mathbb{A}, \mathbb{B}$)	L_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	L_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
8	R($\mathbb{A}, \mathbb{B}$)	R:R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	ToR:R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
9	R($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
10	L($\mathbb{A}, \mathbb{B}$)	L($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	ToL($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
11	R($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)
12	B($\mathbb{A}, \mathbb{B}$)	B($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	B($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
13	R($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
14	R($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)

of time complexity (image 6). M1 uses the concept of both cone-shaped and MBR model at an optimum level, which is explored in Table 5. At this point, the relationship from image 1 demonstrates its behavior. Although it cannot be generalized. Result from image 2 gives its distinctness. Point to region approach in M1 gives more information and this can be revealed by taking images 3, 6, 8 and 10 (Table 5). M2 behaves quite differently than other models (Table 6). Unlike other models, confusion does not retain in image 1. In angle theory, the premier clue is '$\mathbb{A}$' is found to be reference, i.e. '$\mathbb{B}$' *Contain*s '$\mathbb{A}$' and $\mathbb{R}p = c_{\mathbb{A}} = c_{\mathbb{B}}$, it refers to same point. While, in the application of MBR, it presents $\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, Ct($\mathbb{B}, \mathbb{R}g$). This clearly demonstrates its potential towards relationship representation.

Due to very small variation in the shape and size of the components, models do not give perfect distinct relationships but on the other hand, these models gain significantly different relationships for largely varied spatial objects. For such a small variation, angle based theory presents better performance.

On the whole, cone shaped model is best viewed only for approximately similar sized objects at a considerable distance under directional relations. MBR generates sufficient information (directional, topological and metric) compared to cone-shaped model. But the compactness factor limits its efficiency. In such a case, M1 behaves as if angle based theory comes together with MBR. It expresses the idea of dynamic coverage through every corner, looking from the center. All the relationships drawn from M2 are clearly distinguishable with other models. Due to its symmetric behavior, it is easier to reconstruct those configurations.

Table 5: Spatial Information through M1

Image	Point to Point	Point to Region
1	$c_{\mathbb{A}} = c_{\mathbb{B}}$, I($\mathbb{A}, \mathbb{B}$) DC($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
2	B($\mathbb{A}, \mathbb{B}$) below $c\mathbb{B}$, CB($\mathbb{A}, \mathbb{B}$)	B($\mathbb{A}, \mathbb{B}$), CB($\mathbb{A}, \mathbb{B}$)
3	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	ToR($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
4	R($\mathbb{A}, \mathbb{B}$), O($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), O($\mathbb{A}, \mathbb{B}$)
5	T($\mathbb{A}, \mathbb{B}$), I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	I($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
6	R_B($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)	RoB($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)
7	L_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	L_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
8	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	ToR:R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
9	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
10	L($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	ToL($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
11	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), EC($\mathbb{A}, \mathbb{B}$)
12	B($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	B($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
13	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R_T($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)
14	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)	R($\mathbb{A}, \mathbb{B}$), DC($\mathbb{A}, \mathbb{B}$)

Table 6: Spatial Information thorough M2

Image	Use of Ref. Point (Cone-shaped)	Use of Ref. Region (MBR)
1	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, $c_{\mathbb{A}} = c_{\mathbb{B}}$	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, Ct($\mathbb{B}$, $\mathbb{R}g$)
2	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, $c_{\mathbb{A}} = \mathbb{R}p$, T($\mathbb{B}$, $\mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, Cr($\mathbb{B}$, $\mathbb{R}g$)
3	R($\mathbb{A}$, $\mathbb{R}p$), L_B($\mathbb{B}$, $\mathbb{R}p$)	R($\mathbb{A}$, $\mathbb{R}g$), L:L_T($\mathbb{B}$, $\mathbb{R}g$)
4	R($\mathbb{A}$, $\mathbb{R}p$), L($\mathbb{B}$, $\mathbb{R}p$)	O($\mathbb{A}$, $\mathbb{R}g$) (Open-R), Cr($\mathbb{B}$, $\mathbb{R}g$)
5	$\mathbb{A}$-Ref. $\Rightarrow c_{\mathbb{A}} = \mathbb{R}p$, B($\mathbb{B}$, $\mathbb{R}p$)	$\mathbb{A}$-Ref. $\Rightarrow$ MBR($\mathbb{A}$) = $\mathbb{R}g$, Ct($\mathbb{B}$, $\mathbb{R}g$)
6	B($\mathbb{A}$, $\mathbb{R}p$), L_T($\mathbb{A}$, $\mathbb{R}p$)	B($\mathbb{A}$, $\mathbb{R}g$), L_T:T($\mathbb{B}$, $\mathbb{R}g$)
7	L_T($\mathbb{A}$, $\mathbb{R}p$), R_B($\mathbb{B}$, $\mathbb{R}p$)	L_T($\mathbb{A}$, $\mathbb{R}g$) R_T($\mathbb{B}$, $\mathbb{R}g$)
8	R($\mathbb{A}$, $\mathbb{R}p$), L($\mathbb{B}$, $\mathbb{R}p$)	R:R_T($\mathbb{A}$, $\mathbb{R}g$), L:L_T($\mathbb{B}$, $\mathbb{R}g$)
9	R($\mathbb{A}$, $\mathbb{R}p$), L($\mathbb{B}$, $\mathbb{R}p$)	R:R_T($\mathbb{A}$, $\mathbb{R}g$), L:L_T($\mathbb{B}$, $\mathbb{R}g$)
10	L($\mathbb{A}$, $\mathbb{R}p$), R($\mathbb{B}$, $\mathbb{R}p$)	R($\mathbb{A}$, $\mathbb{R}g$), R:R_B($\mathbb{B}$, $\mathbb{R}g$)
11	R($\mathbb{A}$, $\mathbb{R}p$), L($\mathbb{B}$, $\mathbb{R}p$)	R($\mathbb{A}$, $\mathbb{R}g$), L_B:L_T($\mathbb{B}$, $\mathbb{R}g$)
12	B($\mathbb{A}$, $\mathbb{R}p$), T($\mathbb{B}$, $\mathbb{R}p$)	B($\mathbb{A}$, $\mathbb{R}g$), L_T:R_T($\mathbb{B}$, $\mathbb{R}g$)
13	R($\mathbb{A}$, $\mathbb{R}p$), L_B($\mathbb{B}$, $\mathbb{R}p$)	R_T($\mathbb{A}$, $\mathbb{R}g$), L_B($\mathbb{B}$, $\mathbb{R}g$)
14	R($\mathbb{A}$, $\mathbb{R}p$), L($\mathbb{B}$, $\mathbb{R}p$)	R($\mathbb{A}$, $\mathbb{R}g$), L_B:R_T($\mathbb{B}$, $\mathbb{R}g$)

6 Conclusions and Future Works

The paper presented how existing models can be combined for obtaining precise and meaningful relationships, which automatically recover common problems. Those hybrid models are promising in case of regular, crisp and complex within regularity. The processing is quite fast (a couple of seconds) and fixed for any sizes of spatial objects but, PWM is directly affected by the size of objects. The models are faster in comparison since we use PWM only where false connection/overlapping occurs. No one can side line the use of point to point relationship as it comes to be the fastest among all.

A prototype for point to region in all models is extensible to region to point, and region to region under the same scope. In addition, the representation of those complex and irregular objects must be improved by using interest points and that would build true relationships instead of approximation. One of the hybrid models (M2) gives an opportunity to reconstruct those configurations along with worthwile use of metric information. Besides,further investigations will be considered according to Freeman [5, 6]. It shows the way how to use the semantic concept of spatial on components of raster graphical symbols and eventually leads to recognition process and is followed by [2]. As it is possible to use in large queries retrieval, it can be embedded in recognition process. This will be headed as soon as we will have used complex spatial queries in all configurations.

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